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Converting Qualitative Assessments to Quantitative Assumptions: A Bayesian Approach

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Abstract

When developing quantitative scenarios, it is necessary to choose parameter values to fit a particular story line, such as growth rates and resource-use efficiencies. This paper describes one approach to making such choices using Bayesian statistical reasoning to systematically combine data from a reference set (e.g., historical data) with qualitative assessments regarding the scenario. The paper also describes a simple software tool that implements the method.

Introduction

The core of a scenario is the story that it tells, and in any scenario exercise, a qualitative narrative is essential. In addition to the narrative, many scenario exercises are supplemented by quantitative analysis, whether for illustration – a graphical display of the expected trends in the scenario – or for insight – modeling in order to explore the consequences of the scenario narrative. In either case, if quantitative analysis is carried out then at some point it is necessary to translate the qualitative scenario narrative into quantitative parameters. This step unavoidably involves human judgment. Moreover, there is not a unique association between a scenario and a parameter value, and so it is difficult even for experts to pick quantitative parameter values in a consistent way.

The task can be made easier by thinking in terms of departures from a reference point and describing those departures in qualitative terms. Generally, in a scenario exercise, the scenario narrative will be developed before the quantification. By the time quantitative parameters are decided, the people involved in scenario development will have reached some agreement on the qualitative outlines of the scenarios and will have some descriptive phrases that summarize the main points of the scenario (Nakićenović et al., 2000; Gallopín et al, 1997; Hughes and Hillebrand, 2006; Alcamo, 2001). By using qualitative statements that compare the scenario to a reference point, such as, “the rate of economic growth will be like that of China in the 1990s,” or “crop yields will increase at a rate substantially higher than they have over the past decade” the task of specifying parameter values is less fraught and is easier to convey the thought processes that lie behind them.

This paper presents a method based on Bayesian statistical reasoning for generating parameter values starting from qualitative descriptions of how the parameter differs from a reference. The result is a probability distribution for the parameter. By producing a distribution, rather than a single value, the method explicitly recognizes that parameter values are not unique.

Bayesian interpretation of scenario quantification

When considering possible values for a scenario parameter, the person thinking about the values is implicitly asking him or herself what the probability is that the parameter would take on that value in case the scenario in question were to come about. That is, he or she considers the conditional probability

$$P(z|S) = \text{Probability the parameter takes value } z \text{ in scenario } S. \tag{1}$$

However, it is difficult to think about raw probabilities for reasons similar to the reasons why it is difficult to think in terms of specific values for parameters – there are no clear reference points. A reformulation of the conditional probability in Equation (1) makes the connection to a reference point explicit. According to Bayes' rule (Jensen, 1996; Bayes, 1763), the conditional probability satisfies

$$P(z|S) \propto P(S|z)P(z), \tag{2}$$

where $P(S|z)$ is the conditional probability of being in scenario S given z, and $P(z)$ is the prior probability for the parameter. The symbol $\propto$ means "proportional to". The constant of proportionality can be determined by demanding that the total probability is equal to one,

$$\sum_z P(z|S) = 1. \tag{3}$$

Equation (2) recasts the specification of the conditional probability in terms of two factors:

1. A prior probability $P(z)$ that specifies the probability distribution for the parameter in a reference case.
2. The conditional probability $P(S|z)$, which can be thought of as a measure of the degree of confidence that scenario S is unfolding, given the information that the parameter has taken on the value z.

Note that the probability $P(S|z)$ is not the probability that the scenario will come about, a statement that has come to be seen as problematic at best in the scenario literature. An interpretation of the factor will be discussed in detail later in the paper.

Operationalization of Bayesian parameter specification

In order to operationalize the Bayesian interpretation of parameter specification captured by Equation (2), it is necessary to specify a prior distribution for the parameter as well as the probability that the system is in scenario S given the value of the parameter.

The prior distribution

The prior distribution is the probability distribution for the parameter in a reference case. The choice of reference case will depend on the nature of the study as well as the availability of data. For example, when setting an economic growth rate (such as the rate of growth in GDP per capita), historical data from some period may be used. If the reference case is one of relative optimism, then the distribution may be drawn from a set of relatively fast-growing economies over that period. Other choices would be made if the reference case is pessimistic or "trend", in the sense that future performance is expected to look like past performance.

In order to make the prior distribution understandable both the modelers and the stakeholders who generate the qualitative scenario, the reference case should be describable in words that suggest a source for the data. For example, it may be, "the distribution of economic growth rates will be like those seen in Africa over the last quarter of the 20th century". If the time scale of the scenario is, say, a 30-year period, broken into two 15-year periods, then the average annual growth over 15-year periods for African countries at the end of the 20th century could be collected and the observed distribution of growth rates used for the prior distribution.

It is convenient for purposes of elicitation to break the prior distribution into discrete steps that make sense to the people defining the scenarios. A convenient discretization that will be used in this paper is the following one, where q_k indicates the k^{th} quantile (that is, the probability that the value will be less than q_k is equal to k):

$$\begin{aligned} p_{\text{discr}}(z=q_{0.025}) &= 5\% \\ p_{\text{discr}}(z=q_{0.150}) &= 20\% \\ p_{\text{discr}}(z=q_{0.500}) &= 50\% \; . \\ p_{\text{discr}}(z=q_{0.850}) &= 20\% \\ p_{\text{discr}}(z=q_{0.975}) &= 5\% \end{aligned} \tag{4}$$

To connect these values with the mental categories of the people developing the scenarios, each of the values in Equation (4) can be given a qualitative label:

- $q_{0.025}$ is "Very low"
- $q_{0.150}$ is "Low"
- $q_{0.500}$ is "Moderate"
- $q_{0.850}$ is "High"
- $q_{0.975}$ is "Very high"

The quantile values in Equation (4) are set so that they are in the middle of subsequent ranges spanning 5%, 20%, 50%, 20%, and 5% of the full distribution. The example provided above, of economic growth rates in Africa, is shown in Illustration 1, using data on 15-year average annual growth rates in GDP per capita for countries in Africa from 1975 through 2002.

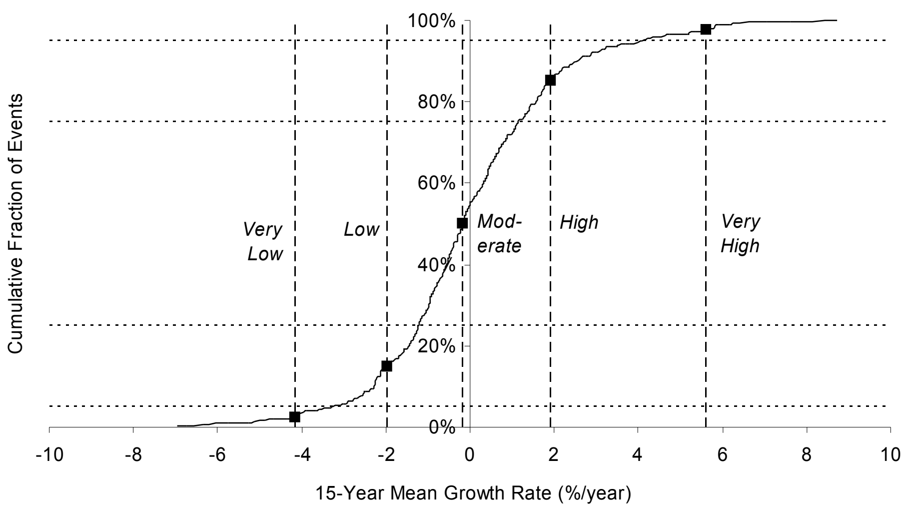

Illustration 1: Distribution and quantiles for growth rates for countries in Africa

The likelihood ratio and the pundit's wager

With the prior distribution specified, the remaining information needed in Equation (2) to generate the probability distribution for the parameter in the scenario is the conditional probability $P(S|z)$. This is the subjective probability that scenario S is taking place, given the information that the parameter value is z. It is useful to think of this conditional probability $P(S|z)$ as a "pundit's wager". It is proportional to the odds that a pundit would give that he or she is correct in saying that scenario S is happening. For example, such a claim might take the form, "Growth has been consistently very high over the past 15 years. It is clear that the revolution in economic fortune that the proponents of scenario S had said could happen is indeed happening." The pundit in this case might place good odds on being correct. Other people may place different odds. For example, if scenario S assumes that liberalized markets will lead to rapid growth, then someone skeptical of market liberalization might say that, given only the information that economic growth is high, it is unlikely that scenario S is taking place. The values must be set in a context of discussion and critical inquiry and be consistent with the scenario narrative.

As with the prior distribution, it is convenient to discretize the values of the likelihood function. The discrete values can then be assigned labels that have a compelling qualitative interpretation. For this paper, the five following qualitative labels are used for $P(S|z)$:

- Very unlikely
- Somewhat unlikely
- Hard to tell
- Somewhat likely
- Very likely

While not strictly necessary, it is convenient to associate the labels with quantitative values by specifying a fixed ratio (the likelihood ratio or odds ratio) between each step in the sequence. This approach leads to reasonable values in practice and simplifies the task of generating the probabilities. Denoting this ratio by R, the probabilities are proportional to the following values:

- Very unlikely: $1/R^2$
- Somewhat unlikely: $1/R$
- Hard to tell: 1
- Somewhat likely: R
- Very likely: R^2

For example, if $R = 2$, then the possible values for the pundit's wager $P(S|z)$ are proportional to ¼, ½, 1, 2, and 4. The values can also be expressed in terms of odds. For example, if $R = 10$ then the sequence would be 100 : 1 against scenario *S;* 10 : 1 against; even odds; 10 : 1 in favor; and 100 : 1 in favor.

By specifying a fixed ratio between steps, the likelihoods follow a logarithmic scale, as opposed to a linear scale, in which values differ by a fixed difference from one step to the next (such as 1, 2, 3, 4, and 5). Logarithmic scales are commonly used in situations where values can vary by many orders of magnitude, as for the decibel scale for loudness, the pH scale for acidity, and the Richter scale for the strength of earthquakes. Likelihoods typically do range over many orders of magnitude and so likelihood ratios and log likelihood ratios are commonly used for statistical model evaluation (Gill, 2002) and in the psychological study of signal detection (McNicol, 2005).

Implementation as a computer program

The procedure outlined in this paper is sufficiently complicated that the support of a software tool can be useful. The BayesScenParams program (Illustration 2) makes the procedure relatively easy to implement. As shown in the illustration, reference data are first entered: these are the data that provide the prior distribution. The data entered in Illustration 2 are the same data used to generate the curve shown in Illustration 1.

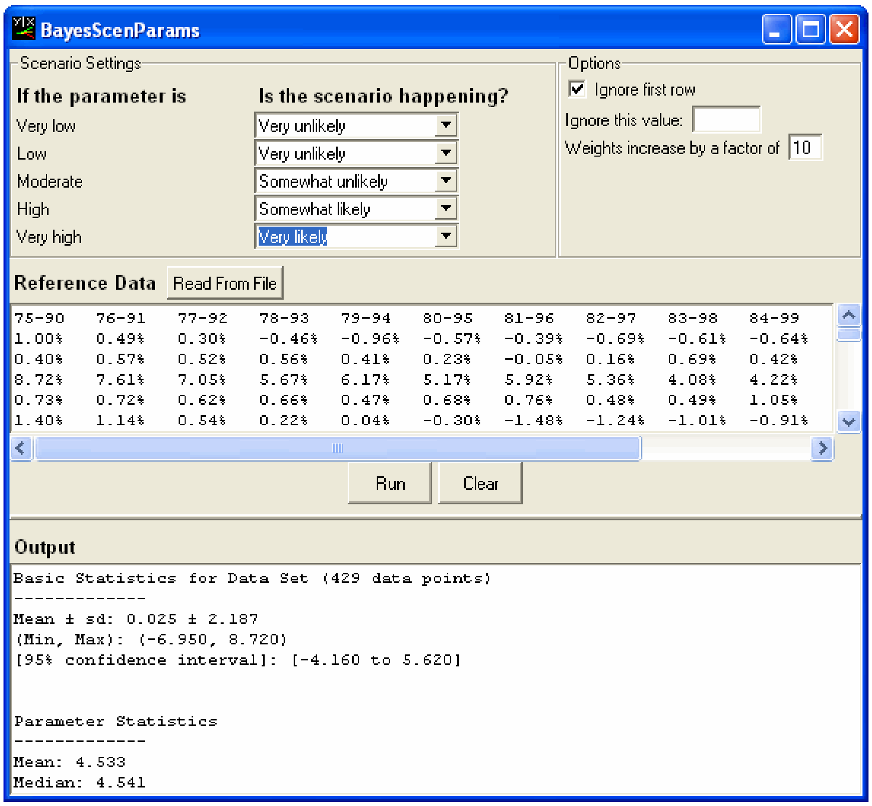

Illustration 2: Screenshot of the BayesScenParams program

As shown in Illustration 2, a high growth scenario is being implemented. This is reflected in the scenario settings, where it can be seen, for example, that the “pundit’s wager” for the case of very low growth is that it is very unlikely that the high growth scenario is taking place, while for the case of very high growth it is considered very likely. As shown in the illustration, the weights are set to increase by a factor of 10, meaning that the ratio $R = 10$. This means that a designation of “very unlikely” translates into an odds ratio of 100 : 1 against, while a designation of “very likely” translates into an odds ratio of 100 : 1 in favor. Turning to the other program options shown in Illustration 2, the first line of the data file contains column labels, rather than data, and so the option “ignore first row” has been selected. There are no missing data, and so there is no need to set a value to ignore.

The output of the program is the estimated mean and median values for the parameter as calculated using the probability distribution $P(z|S)$, as defined in Equation (1). As can be seen in Illustration 2, in the reference data set the mean is a growth rate of close to 0.0% per year. In the alternate scenario defined by the scenario settings, both the mean and the median growth rates are 4.5% per year.

Note that although the method described in this paper generates a distribution, the BayesScenParam program offers two values: the mean and the median. This is for convenience, since in practice only value will be entered into a model. If the distribution is strongly skewed toward low or high values, then the mean and the median will be different. In the case of a skewed distribution, the median is often a better measure of a "typical" value, but even with skewed distributions it is sometimes best to use the mean. For this reason, both are offered.

Conclusions and recommendations

The procedure described in this paper can make the task of generating quantitative scenario inputs from qualitative scenario narratives more systematic. Within a Bayesian framework the task can be seen as a combination of two steps: the choice of a reference probability distribution for the parameter and a specification of how the probability distribution would differ in an alternate scenario compared to the reference case. The inputs to the process are a set of qualitative judgments, expressed as a "pundit's wager"; that is, the likelihood that the scenario is in fact underway given the value of the parameter. Using the BayesScenParams program the implementation of the method is reasonably straightforward, making it possible to apply it in the course of a workshop.

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